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JB-003-001617

Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

August – 2019

Mathematics : BSMT - 602 (A)

(Analysis - II & Abstract Algebra - II)

Faculty Code : 003
Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate full marks of the question.

1 Answers the following questions in short : **20**

(1) How many units in the $(\mathbb{Z}_{10}, +_{10}, \times_{10})$? List them.

(2) What is the multiplicative inverse of 3 in $(\mathbb{Z}_5, +_5, \times_5)$?

(3) List all the ideals of the ring $(\mathbb{Q}, +, \cdot)$.

(4) Let $f : G \to G; f(g) = g(g \in G)$ be a group homomorphism. Find ker f.

(5) True or False : Every integral domain is a field.

(6) True or False : $[0,1)$ is a connected subset of $\mathbb{R}$.

(7) Give an example of a sequentially compact metric space.

(8) Give an example of a non-commutative ring with unity.

(9) If $f = (2, -1, 0, 11, 0, 0, 0, \ldots)$ and
$g = (-1, 8, 2, -2, 0, 0, 0, \ldots)$,
then find $f + g$.

(10) Find $L\left(t^3\right)$.

(11) Find $L^{-1}\left(\frac{s^2 - 3s + 4}{s^3}\right)$

(12) Find convolution product of $f(t) = \sin t$ and $g(t) = t$.

(13) Find $L^{-1}\left(\frac{1}{s-3}\right)$.

(14) State First Shifting Theorem for Laplace Transform.

(15) Define : Sequentially Compact.

(16) Define : Group homomorphism.

(17) Define : Countable set.

(18) Define : Compact set.

(19) Define : Integral domain.

(20) Define : Ideal.

2 (A) Attempt any **three** : **6**

(1) Show that $(0, 1]$ is not a compact subset of $\mathbb{R}$.

(2) Give an example of a closed subset of $\mathbb{R}$ which is not compact.

(3) Show that $\mathbb{R}$ is not a sequentially compact.

(4) Show that $\{1, 2, 3\}$ is a compact subset of $\mathbb{R}$.

(5) Find Laplace transform of $f(t) = \begin{cases} 3 & 0 < t < 5 \\ 0 & t > 5. \end{cases}$

(6) Find $L^{-1}(F(s))$. Where $F(s) = \log \frac{s+a}{s+b}$

(B) Attempt any **three** : **9**

(1) Prove that every totally bounded subset of a metric space is bounded.

(2) Show that $\mathbb{Z}$ is countable.

(3) Show that every compact subset of a metric space is bounded.

(4) Prove that continuous image of a compact set is compact.

(5) Find Laplace transform of $f(t) = (\sin 2t - \cos 2t)^2$.

(6) Find inverse Laplace transform of $F(s) = \frac{3s+7}{s^2 - 2s - 3}$.

(C) Attempt any **two** : **10**

(1) State and prove Heine-Borel Theorem for $\mathbb{R}$.

(2) State and prove Convolution Theorem.

(3) Prove that every compact subset of a metric space (X,d) is closed.

(4) Prove that continuous image of a connected set is connected.

(5) Evaluate $\int_0^\infty e^{-2t}\sin^3 t\, dt$.

3 (A) Attempt any **three** : **6**

(1) Does union of two subrings of a ring is a subring of the ring ? Justify.

(2) Show that in an integral domain 0 and 1 are the only idempotent elements.

(3) Show that $\mathbb{Z}\left[\sqrt{2}\right]=\left\{a+b\sqrt{2} \mid a,b\in\mathbb{Z}\right\}$ is not a field under usual addition and multiplication.

(4) Let $\mathbb{Q}\left[\sqrt{2}\right]=\left\{a+b\sqrt{2} \mid a,b\in\mathbb{Q}\right\}$ is a ring under usual addition and multiplication. Find inverse of $-1+2\sqrt{2}$ in $\mathbb{Q}\left[\sqrt{2}\right]$.

(5) Does $S=\left\{A\in M_2(\mathbb{R}) \mid \det(A)=0\right\}$ is a subring of $\left(M_2(\mathbb{R}),+,\cdot\right)$? Justify.

(6) Does $\left(M_2(\mathbb{Z}),+,\cdot\right)$ the ring of matrices is an integral domain ? Justify.

(B) Attempt any **three** : **9**

(1) Let $f:G\to\overline{G}$ be a group homomorphism. Show that f is one-one iff ker $f=\{e\}$.

(2) Prove that intersection of two ideals of a ring is also an ideal of the ring.

(3) Let R be a commutative ring and $a \in R$.

Show that $A = \{x \in R \mid ax = 0\}$ is an ideal in R.

(4) Show that $(\mathbb{Z}, +, \cdot)$ is a principal ideal ring.

(5) Show that the polynomial

$f(x) = 8x^3 + 6x^2 - 9x + 24$ is irreducible over $\mathbb{Q}$.

(6) Prove that the characteristic of an integral domain is 0 or prime.

(C) Attempt any **two** : **10**

(1) State and prove Fundamental Theorem of group homomorphism.

(2) Prove that every finite integral domain is a field.

(3) Prove that field has no proper ideal.

(4) If F is a field, then show that $F[x]$ is never a field.

(5) State and prove division algorithm in $F[x]$, where F is a field.